Linear Referencing
and Alternate Expressions of Location for Transportation

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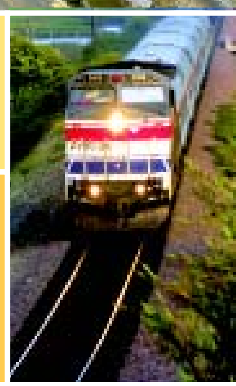


VITAL

Vehicle Intelligence and Transportation Analysis Laboratory
National Center for Geographic Information and Analysis
University of California, Santa Barbara

Linear Referencing and Other Forms of Location Expression for Transportation

California Department of Transportation
Testbed Center for Interoperability
Task Order 3021

FINAL REPORT

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Glossary

AVL	Automated Vehicle Location
Caltrans	California Department of Transportation
DGPS	Differential GPS
DMI	Distance Measuring Instrument
DOT	Department of Transportation
FHWA	Federal Highway Administration, US DOT
GIS	Geographic Information System
GIS-T	Geographic Information System for Transportation
GPS	Global Positioning System
ITS	Intelligent Transportation Systems
LBS	Location Based Services
LR	Linear Referencing
LRMS	Location Referencing Message Specification, SAE Information Report J2374
LRP	Linear Referencing Profile (part of LRMS)
LX	Location eXpression
NCHRP	National Cooperative Highway Research Program
NMEA	National Marine Electronics Association
S/A	Selective Availability. Deliberate degradation of GPS signal by U.S. Department of Defense. The practice was discontinued in May 2000. GPS coordinates were expected to be accurate within 100 m with S/A turned on, and about 10 m with S/A off. DGPS improves accuracy in either case.
SAE	Society of Automotive Engineers
SWITRS	State Wide Integrated Traffic Record System
UCSB	University of California, Santa Barbara
UTM	Universal Transverse Mercator
VITAL	Vehicle Intelligence and Transportation Analysis Laboratory, UCSB
WAAS	Wide Area Augmentation System, a Federal Aviation Administration inspired GPS differential correction system and enhancement, which yields accuracy of ±3 m on suitably equipped receivers (*www.garmin.com/aboutGPS/waas.html*). The WAAS satellite infrastructure is not yet well developed.
XSP	Cross Streets Profile (part of LRMS)

Introduction

There are numerous everyday situations in which it is necessary to express a location on a transportation network, e.g. emergency reporting and highway maintenance. Describing a location is not as simple as it might seem. Coordinates may not work well, street addresses do not always exist on highways and landmarks may not be known to both communicating parties. Highway professionals have traditionally employed Linear Referencing (LR), expressing a location as a distance from a known starting point in a given direction; however, the need to measure distance in the field accurately makes this a laborious and error-prone process. With new technologies such as GPS now available, and user expectations of accuracy greater than ever before, there is a need to re-examine current practices in Location Expression (LX; LR is one form of LX), and to develop methods that take advantage of new technologies, while maintaining compatibility with legacy systems.

The authors are tasked under Caltrans-UCSB Task Order 3021 to examine LR and other methods of LX for use in transportation management agencies such as Caltrans, for applications including Intelligent Transportation Systems (ITS) and pavement management. This study dovetails with previous FHWA-sponsored work, evaluating the Location Referencing Message Specification (LRMS) Linear Referencing Profile, a proposed standard for LX, in an ITS applications context. There is also a link with ongoing research on remote sensing applications in transportation, in particular the derivation of road centerline geometry from satellite/aerial imagery and GPS — these links are elaborated in the body of the report.

A useful starting point for this study is the publication "Linear Referencing Practitioners Guidebook" (FHWA 1997), which documents the practice of LR with reference to a small sample of state DOTs. The book presents the problems of LX and discusses how LR must be implemented. On the other hand, it was written before the withdrawal of "Selective Availability" of GPS, and for that and perhaps other reasons, understates the potential of GPS as a viable alternative to LR.

LX technologies are evolving rapidly — the cost of Inertial Measurement Units (IMUs) is decreasing, and novel techniques such as multipath signal mapping are emerging — and it is inevitable that LR will increasingly share the stage with other LX methods. There is no way to predict whether or when GPS or any other technology will overtake LR; the point is not the outcome of a competition as much as it is the need for these and future methods to co-exist, for the sake of (a) compatibility with legacy databases, and (b) data sharing.

Specifically, we are concerned with the accuracy required by transportation professionals when specifying a location. Should a linearly specified location be recoverable accurate to ±1 m, ±10 m or ±100 m? This is clearly application-dependent, but for pavement and asset management, the number most often quoted is 15 m. To achieve this with Distance Measuring Instrument (DMI) technology, which has a measurement error of about 1%, one requires a network of established reference points so that proportional errors in measurement can be controlled. Such a network is a form of infrastructure, and needs to be planned, established, documented and maintained. GPS measurement requires no such infrastructure; on the other hand, for a GPS reading to be meaningfully interchanged with a linear measurement, it must be referenced to a geometrically accurate representation of the road centerline, the acquisition of which is also a substantial infrastructure-type undertaking. What is the geometric accuracy required of a centerline so that linear and GPS-based measurements can co-exist? How can such a database be developed to meet the criteria of "faster, cheaper and better?"

This is an overview report that synthesizes the principal conclusions from the FHWA-sponsored ITS standards project, specifically the LRMS Linear Referencing Profile. It presents the principal results of that work, ties it to ongoing research, and outlines the principal conclusions and outstanding issues.

Linear Referencing

Linear Referencing (LR) is the specification of a location by means of a distance measurement along a sequence of road sections (a "traversal") from a known reference point (the "traversal reference point"). The phrase "25 km west of Santa Barbara on US-101" is a valid linear reference, but for highway management applications the resolution of a LR would normally be much finer, e.g. "738 m west of traversal reference point 496.25 on US-101 Northbound." The traversal reference point is documented and recoverable in the field; it may be an intersection, water tower, fire hydrant, or a marker such as a numbered pole embedded in the ground. Distance is measured along the road and therefore reflects curves, elevation and unevenness in the road surface.

Measuring Distance

When LR relates points on the ground to records in a database, it relies on two measurements: one is the algebraic computation of the length of a digital polyline or geometric arc, the other is the measurement of distance in the field. The former task is easily handled using a Euclidean distance calculation over each segment of the polyline, or an appropriate algebraic solution in the case of a geometric arc. The latter task, measuring distance in the field, is accomplished by various technologies, and many types of error can be introduced. The following is a brief overview of the technologies.

Rangefinders

A rangefinder employs a beam of energy (e.g. laser) that reflects off an object, and calculates distance based on the signal return time. This gives the straight line distance in the field. This is usually not the appropriate measurement for transportation, where the task is to obtain driven road distance, not straight line distance.

Distance Measuring Instruments (DMIs)

A DMI is essentially a high-resolution odometer, that enables road distance to be measured simply by driving a vehicle over the road. DMI technology has evolved over the years, from mechanical revolution counters on a "fifth wheel" to optical sensors that count revolutions of a vehicle wheel, and most recently, electronic pulse counters attached to the vehicle transmission.

The modern (pulse counter) DMI is supplied with a sensor cable that must be spliced into the vehicle's transmission system. In the passenger compartment, a calculator-like unit translates the pulses into a distance measurement (the instrument must be calibrated to set up this translation). Command buttons on the unit start, stop and reset the counter. On more advanced systems, the display can be interfaced to a computer, or the DMI can be programmed to control an external device such as a paint striping system. The $500 Nu-Metrics Nitestar is one of the most capable and popular units on the market, and comments in this report are based principally on observations with this model.

Types of Measurement Error

Several types of error *can* be present in a linear reference. All errors are not always present, and clearly the more experienced professionals go to great lengths to minimize error. Errors can be categorized into (a) terminal error, that occurs at one or more points along the traversal, and (b) proportional error, that accumulates over the traversal.

Terminal Error

Operator Cueing Error

When the measuring vehicle is traveling at normal traffic speed, the accuracy of recording the start and end point is only as good as operator reflex. Errors of 0–10 m are possible. Much depends on the visibility of the target point: a tall light standard located to the side of the vehicle path is easier to sight accurately than is a spot on the ground.

Endpoints: Correct Intersection?

Street intersections may be used as traversal reference points. They are often specified by the names of the intersecting streets. Street names are not always accurate in databases, and not always posted. There are sometimes multiple instances of a street name (as on residential crescents, and sometimes due to non-standard naming practices). Therefore measurement of distance, or recovery of a specified point, may begin at the wrong location.

Endpoints: Where in the Intersection?

A large intersection can be 30 m across, and unless further detail is provided on where in the intersection the starting point is, terminal error can be introduced.

DMI Error

DMIs have mechanical limitations. At low speeds (about 5 km/h and below) it is possible that a DMI records no transmission pulses. This makes it unsuitable for use in heavy traffic or where there are frequent stops for lights and stop signs.

There can be a short latency between the instant the recording button is pressed and the moment the instrument records and processes the request. When requests are handled through a serial interface to a computer, we have observed latencies as long as 1/10 second. If vehicle speed is constant during the drive, this error cancels out.

Proportional Error

DMI Calibration Error

DMIs have good *repeatability*, of 1 in 1000 or better. This is the ability to produce the same reading over several drives of the same course; it is not the same as *accuracy* — that depends on calibration. To calibrate a DMI, one has to drive a measured course — manufacturers recommend a course in the 300–500m range; however, the methods to measure such a course are themselves subject to error. Alternately one could run the drive wheels of the vehicle over carefully engineered steel rollers, and count the revolutions of the rollers. Commercial services that calibrate odometers using such equipment (e.g. Fleet Parts & Instruments, Bakersfield) report that the steel rollers suffer wear and must be re-calibrated about every month; the method of calibration is, ironically, a road course that is measured using a laser rangefinder and *assumed* to be perfectly straight. Fleet promises accuracy of no better than 1%, and this is generally the accepted uncertainty in any DMI measurement.

Line of Measurement

It is difficult to define the exact line that is to be measured, particularly when lanes begin and end along a traversal. Clearly from a scientific standpoint it is best to stay in the same lane throughout the traversal as far as possible. On the other hand, the effect of a lane change is slight: a single (3.6 m) lane change takes place over a distance of at least 50–150 m, depending on vehicle speed. Modeling vehicle path as a sine curve, it is possible to compute the net difference in path length to be 0.1–0.2 m.

Vertical Undulations

Imperfections and undulations in the road surface cause imperceptible errors. A rise and fall (one complete cycle of a sine curve) of amplitude 1m in a 500m road increases length by 13mm or 0.003%.

Driving a Straight Line

It is practically impossible to drive a vehicle in a "perfectly straight" line, yet any deviation from a straight line increases the distance reading. Based on the calculations in the previous paragraph, one can conclude that with reasonably careful driving, the effect of wandering from a straight line is insignificant.

Other Factors

Air and road temperature and tire pressure affect the radius of a tire, and can distort measurements slightly. This is generally not a significant concern. Modern tires have strong side walls, and as long as temperature and pressure are kept reasonably constant (±20°C and ±30 kPa), we have observed the effect on a DMI reading to be less than 0.5%.

Practical Issues

The field measurement aspect of LR is laborious. It requires lane discipline and speed control (minimum speed about 5 km/h) to measure distance with a DMI, and a trip cannot be interrupted without precipitating a series of intermediate notes and calculations. If there is doubt about whether or where the distance counter was reset, the entire stretch of road may have to be driven again.

Relative LRs

Due to the potential for proportional error, LRs are often expressed in relative terms, i.e. a location is specified as the percentage of a traversal length. While this is good practice from the viewpoint of accuracy, it requires that two linear measurements be taken to specify a location.

The Human Factor

Perhaps the greatest source of inaccuracy in LR is human error. Finding an accurate LR is labor-intensive, particularly if relative LRs are used; as a result, in non-critical situations, LRs are sometimes estimated rather than measured. For example, a police officer recording an accident is much more likely to guess that an incident is 0.5 km from an intersection, or simply to note the nearest intersection, than to measure the distance with an odometer (unless the accident is serious, in which case a precise survey may be carried out). Consequently, many accident databases (e.g. California's State Wide Integrated Traffic Record System, SWITRS) show a predominance of accidents at intersections, whereas careful analysis shows that mid-block collisions (e.g. at curb cuts at shopping centers and driveways) account for about 50% of all accidents (Veeramallu et al 2001). From conversations with transportation professionals in the U.S. and Europe, we know that such generalization routinely occurs in DOTs too, and the accuracy of any given measurement is largely dependent on judgement of its importance.

Sharing and Converting Measurements

Practical analysis requires sharing of data that were measured or recorded differently. Linearly specified data (e.g. accident locations) have to be reconciled with coordinate-based data (e.g. susceptibility to fog). Even when the same LX method is used, positions may be difficult to reconcile, e.g. LRs taken by two different agencies may be so different that substantial work is involved in translation.

Linear to 2-dimensional

A linear reference is readily converted into coordinates, by reference to a road centerline. Euclidean distance is accumulated along successive segments of the polyline until the linear measure is attained; over the final segment, the coordinates are interpolated.

2-dimensional to linear

Conversion from 2-dimensional to linear expression requires that the given point be relocated or "snapped" to the nearest segment on the road centerline (the algebra for snapping is described by Sproull et al 1985) although this may not be appropriate if either the coordinate or the centerline contains substantial error, because the point may snap to the wrong segment, as illustrated in the "omega" pathology, Figure 1. Euclidean distance is measured along the polyline until the computed point on that nearest segment is encountered.

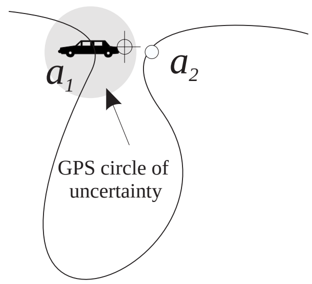

Figure 1. "Omega" pathology: a GPS point intended to snap to a_1 may snap to a_2 instead

Linear↔2-d Conversion accuracy

The above operations assume that there is a road centerline along which length can be measured accurately from coordinate geometry. This is often not the case because historically, the technology for surveying centerline geometry has been inadequate or too expensive to support mapping at the required scale. Consequently a centerline database may under-represent road length by as much as 15%, and coordinate error at any given point or segment may be as high as 200 m. With GPS and photogrammetric methods now more affordable than in the past, this is changing. This issue is revisited later in this report.

Failure Modes

Due to errors in centerline geometry, the following failure modes are possible in the use of linear expressions, or the conversion between linear and 2-d expressions:

Significant linear error

When two different sections of a road come within close proximity of each other (e.g. the omega pathology), there may be a significant error in a linear conversion as illustrated above. Similarly, converting a linear reference to a 2-d coordinate using a poor quality centerline, the derived 2-d position may be incorrect, and in subsequent conversion back to a linear measurement, the error is propagated.

Linear measurement overshoots cross street

Due to measurement error, a location just short of an intersection may be erroneously recorded as falling beyond the intersection. This is potentially a serious error; the likelihood and severity are entirely context-dependent and difficult to quantify.

Other Problems and Solutions

Paradoxically, the problems of sharing and converting linear data are about as difficult as those of converting between 2-dimensional and linear data. A body of physical infrastructure and computational techniques has had to be developed to support LR.

Reference Markers

LR is a relative method of measurement, not an absolute method, therefore it requires a significant field infrastructure of engineering stations and markers that must be planned, installed, documented and maintained.

Alignment Changes

When a road alignment is changed, downstream measurements are invalidated. Therefore agencies must maintain metadata on the date of observation of each linear reference, and must follow-up each change of alignment, either by means of one-time processing of all recorded LRs, or by on-the-fly use of equivalency tables or "mileage equations." This process is not always well managed (FHWA, 1999). Moreover, due to the need to process legacy data periodically, it is inevitable that errors are introduced in the numerous operations required to trace re-alignment histories.

Lateral Offset

LR is 1-dimensional in a multi-dimensional world. It models the location of objects as if they lie exactly on the road centerline. To record the location of off-road "furniture" such as speed limit signs, it may be necessary to measure a lateral offset from the centerline. This requires that the location of the centerline be clearly understood and documented, e.g. a yellow line or Jersey barrier. When the field position of a centerline is uncertain and undocumented, e.g. in the case of a landscaped median, lateral offsets are meaningless.

Best Practice

There are several situations (discussed in FHWA 1999) where the practice of LR is not clearly defined and there are differences in practices between agencies. How does one deal with a divided highway when a traversal exists along one carriageway but not the other? What special rules are required for ramps and rotaries? Measurements are taken from different reference points, along different traversals.

There are variations in practice within agencies and more so between agencies, that inhibit data sharing. This is partly because although LR is a well established practice in DOTs, the computer/GIS aspects of LR emerged quite suddenly during the 1980s and were not standardized. Several studies were commissioned in the 1990s to address this.

- The GIS-T/ISTEA Pooled Fund Study (Fletcher et al 1995) laid out a model for integrated multi-modal, multi-jurisdictional transportation management. The study ran in two phases, and there was a shift in its aims. Although not exclusively focused on LR, it was a significant event in its evolution, drawing attention to its potential and limitations.
- NCHRP project 20-27(2) (Vonderohe et al 1997; Adams et al 1996) established a data model and standard terminology for LR. This project is widely regarded as having made great strides in promoting standardization. Most significant among the recommendations was a network of anchor points and anchor sections to support accuracy. Anchor points are recoverable locations in the field, that are common to all LR systems and thereby tie the systems together. Anchor sections are lengths of road between anchor points, with authoritative linear measures documented. In some respects the model in its original form is too elaborate and rigid for small agencies, and this is one factor that has hindered adoption. For example, to relate events measured using one LR method with those measured using another method, common points ("tie points") need to be established that are common to both methods. Ideally the anchor point network serves this function, but in practice ad-hoc points are required in addition to the formal network; there is no provision for these in the 20-27(2) model.
- An associated study (Vonderohe & Hepworth 1996) established rigorous algebraic optimization procedures to ensure LR accuracy, e.g. setting out the maximum acceptable length of an anchor

section to achieve a given accuracy standard. The study does not establish numeric accuracy standards; these are specified on a state by state basis in DOT operations manuals.

- The FHWA publication referenced above (FHWA 1999) attempted to document practices and to standardize procedures nationwide. It is reviewed in the next chapter.
- Most recently, NCHRP project 20-47 (in progress) was commissioned to study errors in translating between linear and other forms of location expression.

Concluding Points

Linear Referencing clearly has a strong tradition as the preferred means of LX in DOTs. However, there are problems associated with field measurement, and it requires an elaborate field infrastructure and a methodological support structure to achieve current accuracy standards. Accuracy requirements clearly vary depending on the application, but the most often quoted tolerance is 15 m.

The FHWA Linear Referencing Guidebook

This chapter is a brief review of the Federal Highway Administration (FHWA 1999) publication "Linear Referencing Practitioners Guidebook." Generally speaking, the Guidebook is well researched and thorough. It outlines the basis of linear referencing, with useful illustrations on control sections, reference points and other LR constructs. It examines four states in detail (Washington, Idaho, Missouri and Pennsylvania), describing the difficulties faced by DOTs with systems as they evolved, and how they adapted to them with technical solutions and business rules.

Structure

The introductory chapter states the objectives of the book, the intended audience, and how the four case study states are selected. Next the Guidebook offers an overview of LR, covering basic terminology and methods. It presents material in plain English, without the detailed illustrations and calculus found in technical documents on LR. The third chapter describes the case studies in detail, showing how the four subject states have interpreted and applied the theory of LR in different ways, beginning at different times, and what software they use (a) to determine and to record locations and (b) to visualize and to analyze the data.

Chapter 4 discusses LR and GIS. It explains the difference between LR and geodetic (coordinate) referencing, and shows how LR is implemented in GIS. Again the case studies are used as illustrations. It outlines the technique of dynamic segmentation and the problem of conflation.

Chapter 5 is a detailed description of pathological cases and how to handle them. It covers divided highways, ramps, layered roads, rotaries and cul-de-sacs. There is a brief treatment of accuracy and quality control and management of historical data. Chapter 6 covers technologies (GPS, videologging, straight line diagrams, data warehousing) and the Federal Geographic Data Committee's (FGDC) Spatial Data Transfer Standard (SDTS) — one that has not been widely adopted. The Guidebook concludes with a survey of research in LR (NCHRP 20-27(2) and the GIS-T Pooled Fund study) and a summary of findings.

Critique

As a general purpose review of the field, the book is a useful resource, and for those who are committed to employing linear referencing to the exclusion of other methods, it is a good introduction. However, to the extent that it may serve as a Guidebook for all LX, it suffers from some limitations.

1. The Guidebook will be used by DOT personnel to understand current practices, and also to assess options for the future. It does not offer a forecast of user needs (users being both automobile users and DOT personnel), or of the technologies that are already being developed to meet them. Although Intelligent Transportation Systems (ITS) are mentioned, the implications of new technologies on current practices are not discussed.

2. The Distance Measuring Instrument (DMI) is a central equipment item in LR. DMI technologies, field measurement procedures, errors and precautions are not covered.

3. While GPS is mentioned, it does not get the depth of coverage that it deserves. Given that in-vehicle navigation systems widely employ GPS in preference to linear methods, and that many DOT field observations are now GPS-based, it would seem appropriate to present LR and GPS as equal alternatives.

4. The arguments for LR over coordinates are largely based on the discrepancy between driven distance and 2-d computed distance. There is a widely held myth that this discrepancy is caused by differences in elevation. The Guidebook adds to the confusion on this point. It makes frequent references to the effect of elevation, and at one point an illustration shows a 10% difference between 2-d and 3-d distance (discrepancies of this magnitude do *not* arise this way). Then in an apparent afterthought, a short 3-line paragraph effectively retracts the argument, admitting that in practice the difference even on an unusually steep 10% grade is less than 0.5% (this is true). It is true that in the 1:24,000 scale base maps in use at many DOTs, there is a substantial discrepancy between driven and computed distance. But this is the result of horizontal generalization (expanded in the next chapter) and has almost nothing to do with elevation.

Concluding Comments

In short the Guidebook is a valuable resource for those unfamiliar with LR, and in terms of its title it fulfills its mission well. However, there is a danger in treating the Guidebook as the exclusive authority on LX. It is by design focused on LR, not the wider problem of LX. In that respect there is a danger that it may perpetuate past practice, rather than look forward to the needs of DOTs as they deal with a more demanding future, with greater public expectations and updated technology.

Location Expression by Coordinates

This chapter briefly introduces a few notable aspects of coordinates as a means of LX, examines their principal drawbacks for use in highway management, and discusses how the difficulties can be resolved. Coordinates will be compared and reconciled with LR in a later chapter.

General Characteristics

Coordinates express location relative to a global reference system, and this has two important implications:

1. by comparison with linear measurement, the reference system is stable, and
2. coordinates can be used to tie together seemingly unrelated objects, surveyed independently and stored in different systems (e.g. transportation networks and flood susceptibility zones).

Because of these significant advantages, one can argue strongly for all location data to be either coordinate-based, or at least expressible in coordinates. On the other hand, it takes a support system of computers and detailed map databases to tell whether a point expressed in coordinates (34° 23′ 53″ N, 119° 28′ 07″ W) lies 8 m or 8000 m down the road from a given reference point, and for transportation applications this is a major drawback.

Coordinates are scalable, in that an object can be represented at different levels of detail. For example, a bridge can be shown as a point at 1:100,000, a line segment at 1:5,000, and a rectangle of pavement at 1:500; similarly a freeway can be a single centerline running down the median, or a pair of directional carriageways, or centerlines of individual lanes; and a rotary can be treated in terms of its explicit geometry, or generalized into a point intersection. Scalability is both an advantage and a disadvantage. The benefit lies in the flexibility and appropriateness of the representation; the disadvantage is that it leaves room for inappropriate representation — in LR too, difficulties arise when dealing with some of these pathologies.

Problem Areas

Reference System

There are several reference systems on which coordinates can be specified. The principal source of discrepancies between agencies is the reference ellipsoid, which was redefined and standardized in the early 1980s, resulting in the North American Datum of 1983 and the World Geodetic System of 1984 (NAD83/WGS84). Some legacy data still exist in the older NAD27 datum of 1927, but the two datums are well defined and translation routines are now available to exchange data with acceptable precision.

Geometric Accuracy

A more serious problem is geometric error. VITAL researchers tested six commercial street databases using differential-GPS, and found worst case horizontal errors of 200 m (Funk et al 1998), enough to place a vehicle on the wrong street in an Automatic Vehicle Location (AVL) application. Errors were worst:

- in remote areas
- on sinuous roads rather than straight roads (this usually correlates with hilly, remote areas)
- in the geometry of highway interchanges
- in new residential subdivisions.

In older, established street systems such as downtown grids, error was about 5 m.

Linear Accuracy of Coordinate Polylines

In a previous chapter we found that distance measurement along a road (using a DMI) is subject to uncertainty of about 1%. By comparison, computation of road length from centerline geometry using street databases is far more error-prone, with discrepancies up to 15% on more sinuous roads. The ~15% error is *almost entirely* because the databases were developed for smaller scale (e.g. 1:100,000) use, and therefore have a lower density of shape points, resulting in shorter computed length (Figure 2). Clearly the problem is much more severe on winding roads than on straight sections.

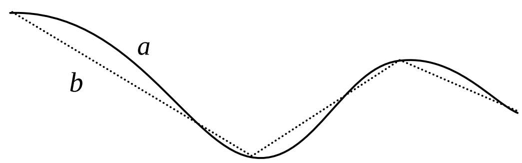

Figure 2. Curve *b*, with a lower density of shape points than *a*, is shorter. This is the real reason why the length of hilly roads (which tend to be sinuous) is usually under-reported.

This raises the question: what is the appropriate map scale — and horizontal accuracy — at which centerlines should be surveyed and mapped to ensure accurate computation of road length from centerline geometry. This is an area of ongoing research at VITAL. In general we have found that centerlines surveyed with differential GPS and even uncorrected GPS can produce better than ±1% linear accuracy; this is documented later in this chapter.

Creating Centerline Databases

The Past

Coordinates have historically been difficult for the average person to derive or to interpret. Because the instrumentation and skills required to find coordinates were so involved, it was difficult to produce accurate maps, and to test maps for accuracy. Many of the better local surveys were developed in planar coordinates, without tying them to any wider spatial reference (e.g. Latitude/Longitude, UTM or even State Plane). Hence it was difficult to mosaic detailed maps for a large area such as the U.S.

In the 1970s the Bureau of the Census used US Geological Survey topographic maps and other sources to produce a nationwide street database, to assist enumerators with the 1980 and subsequent censuses. Their DIME and TIGER products spawned a number of commercial street databases and applications. Although the data were valid only at scales of about 1:50,000 to 1:100,000, they were used for transportation management, vehicle navigation and other applications that required 1:10,000 to 1:25,000 accuracy. When GPS became affordable to consumers in the 1990s, there was both a need for better geometric accuracy and an independent means to test for it. This led to the VITAL accuracy tests reported above.

The Future

There are now a variety of technical approaches to the development of accurate centerline geometry, mostly centered on GPS, photogrammetry and remote sensing.

- To a limited extent, centerlines are derived from photogrammetry. This is expensive, and usually the photography is obtained to facilitate a construction project (e.g. cut and fill calculations) and the centerline is a by-product. Photogrammetry is rarely if ever pursued solely for centerline extraction.
- GPS is used on videologging vehicles, gathering coordinate data in association with ground-level photography, road condition, etc. Commercial firms such as Transmap and Roadware offer equipment and services.

- GPS tracks from multiple vehicles in a fleet can be resolved into centerlines — this is being investigated at VITAL.
- High-precision GPS is being explored for guidance of snow plows and autonomous vehicles. Intelligent Transportation Systems (ITS) are motivating new standards for accuracy at the centimetre level (e.g. Farrell et al 2000), and both GPS and other sensing technologies developed for ITS may be deployed for centerline survey (Shields 2001).
- Remote sensing is being explored as a solution. VITAL leads the National Consortium on Remote Sensing in Transportation—Infrastructure Management, and its principal project is to study the extraction of road centerlines from imagery (VITAL 2001). This is an area of increasing interest, and VITAL, in conjunction with NCRST-Infrastructure, hosted a specialist meeting on centerline technologies in August 2001, with participants from Australia, Canada, New Zealand and the U.S.
- Data can be integrated from municipalities, utilities and other sources. Historically this has been difficult due to differences between CAD and GIS data models. VITAL recently worked with ESRI, a major GIS vendor, to develop the Unified Network for Transportation (UNETRANS) data model that, among other things, facilitates data exchange. UNETRANS is a core set of essential and familiar transportation elements (e.g. roads, carriageways, intersections, bike paths), organized into hierarchical object classes; the UNETRANS handbook documents the model and illustrates how special cases may be handled. Although it is not intended to be a standard, users with fundamentally similar object models will be able to exchange data more readily than in the past, when data models and representations were entirely different. Details on UNETRANS are available at *www.ncgia.ucsb.edu/vital/unetrans*.

$250 GPS hardware can capture centerline geometry with sufficient accuracy for most applications.

GPS

The principles of GPS are widely described in technical literature and web documents (e.g. *www.trimble.com/gps/fsections/aa_f1.htm*; *www.colorado.edu/geography/gcraft/notes/gps/gps_f.html*).

From the standpoint of highway management, GPS has impacted LX in two important ways:

1. it has improved the ability to create accurate centerline geometry;
2. it has made it possible for ordinary citizens — and highway personnel with no special training — to report position in coordinates.

There is a variety of GPS hardware available, from $125 handheld units sold at retailers such as CompUSA and Staples, to specialized survey-grade receivers costing several thousand dollars, that promise cm-accuracy positioning and integrated inertial measurement. One could argue that with new applications demanding coordinates for lane stripes at centimetre accuracy, DOTs are well advised to plan for detailed road infrastructure surveys with high-end equipment. This is certainly one option, appropriate on well traveled roads where more exacting ITS applications are likely to be deployed.

In remote areas where agencies currently have no geometry or poor (±50 m) databases, and cannot afford more accurate data, consumer grade GPS now offers an opportunity to get started effectively at the low end. The $125 Magellan GPS 315 and the $150 Magellan GPS Companion can capture centerline geometry with sufficient accuracy for many applications. To record large volumes of data as required for centerline geometry, the above units currently require a live interface to a handheld or notebook computer (future firmware enhancements may obviate this). The Companion is designed specifically with a host computer in mind: it slips into the expansion slot of a $200 Palm or Visor handheld computer, which can log several days' worth of GPS data. The portability of this combination is compelling, and one obvious potential application is in fleet vehicles, to capture both centerline coordinates and drive time. Two handheld units in the $250 range — the Magellan MAP 330 and Garmin eTrex Legend — offer Wide Area Augmentation Service (WAAS) correction. WAAS is a Federal Aviation Administration inspired correction system, intended to improve aircraft navigation in poor visibility, but is free to those with suitably equipped receivers. It claims ±3 m accuracy, but currently the signal is broadcast from just 2 geostationary satellites in the U.S. and is not always available.

GPS Data and Linearly Accuracy

It is possible for digitized coordinates to stray from the truth individually or collectively, and yet achieve linear accuracy. For example, the Thomas Brothers digital database (as tested in 1997) is inaccurate by about ±200 m in places, but its curves are smooth and the geometric "detail" results in linear measures much closer to DMI distance than with other, more spatially accurate but more sparsely sampled databases. In general, however, road length is most likely to be accurately represented (within ±1%, the accuracy of a DMI) when coordinates are accurate to about ±2–5 m.

Conversely, coordinates could theoretically be within tolerance individually, yet line length could be misreported if the error vector at each vertex is independently oriented. When a line is manually digitized or scanned, the vector azimuths are independent; using GPS, they tend to be persistent. One step in preparing a geometric database is to weed out redundant (within-tolerance) vertices on straight sections of road, using popular algorithms (e.g. Douglas & Poiker 1973). Such algorithms require as input a lateral displacement tolerance (LDT). The relationship between LDT and linear accuracy needs to be explored in the context of LR, and numerical values need to be established to relate coordinate accuracy, linear accuracy and LDT.

Recently VITAL conducted tests in which consumer grade GPS units were simply suspended from a rear-view mirror, to record centerline data while driving — a low-budget centerline survey. The trajectories were post-processed using line generalization as described above, and curve-fitting was applied on sharp bends to interpolate between the 1 Hz GPS data. Linear measures were compared against DMI readings. The results were surprisingly good: on a winding mountain road, a $150 GPS produced centerlines with linear measures within 0.5% of the DMI. Post-processing added about 0.25% to polyline length. Significantly, the GPS lengths were more accurate and consistent than those of the DMI on steep (7–8%) uphill runs, where the DMI is apparently subject to instrument error, perhaps due to high engine RPM.

Further research remains to be done in this area, but it clear that for routine ±15 m field positioning, e.g. to record the location of potholes and bridges, consumer grade GPS units are sufficient for both centerline database development and for field observation of location.

Conclusions

Early in this chapter we stated that for coordinates to become the basis of LX, they would require a "support system of computers and detailed map databases." The hardware now exists in the form of PDAs, and the next step, the wearable computer (*www.xybernaut.com*), is only a cost step away. The other element in the support system — detailed databases — is currently missing in many DOTs, but the technology for cost-effective solutions is now on the horizon.

Minnesota DOT recently invested $1.2 million to convert all linearly referenced LXs in its archives to coordinates (Gorg, 2001). Other state DOTs are acquiring 2-d and 3-d data to varying degrees, sometimes in cooperation with local governments. These initiatives show the increasing emphasis on 2-d data for compatibility with non-linear data sets. This chapter has argued that these projects can be realized with minimal expenditure.

Findings from LRMS Testing

This chapter is a bridge from the previous discussion on LR and coordinates, to the use of linear and/or coordinate methods as required, to achieve the objective of accurate and unambiguous LX. It summarizes results of tests on proposed standard messages for ITS location referencing (Noronha et al 1999; VITAL 2000). This work was carried out in 1997-99, funded by FHWA and Caltrans, under contract to Oak Ridge National Laboratory. The messages tested were the Cross Streets Profile and the Linear Referencing Profile, taken from the Location Referencing Message Specification (LRMS), subsequently approved by SAE as Information Report J2374.

Cross Streets Profile

The Cross Streets Profile (XSP) is a formal expression of location without the use of coordinates. Assuming that a point lies on a street network, the XSP describes the location in terms of the street segment on which the point lies, and the distance from a segment end-point, i.e. {on-street, from-cross-street, to-cross-street, distance from from-cross-street}. Streets are identified by names. Testing of the profile hinges on interoperability between map databases, i.e. the challenge is to recover accurately and unambiguously a location specified with respect to one map database, against a different map database of the same area.

To test the profile, we generated a large sample of points (10,000 in the County of Santa Barbara) along a source network database, generated a message using the components of the XSP, and interpreted the message with respect to the target database. A large proportion of the street database records — 19% in the best case, 44% in the worst case — had no names. Therefore our tests using the original XSP yielded extremely poor results, with success rates in the region of 5%–25%.

In the meantime, the XSP has seemed so intuitively correct that it was being proposed as the basis for messaging standards at the International Organization of Standards (ISO). We proposed that the message be strengthened by including a coordinate reference, in the hope that the combination of names and coordinates would perform better. Performance was much better, in the 50%–75% range, but still not good enough for mission critical applications.

Linear Referencing Profile

The LRMS Linear Referencing Profile (LRP) expresses a location in terms of driven distance (measured either in the field or by coordinate geometry from a database). The starting point for the measurement is expressed as the intersection of two streets, each specified by numeric identifiers. First, this suggests that the profile is intended for internal communication within an organization, where numeric identifiers are assigned and agreed upon. In reality it is extremely difficult and laborious to accomplish this step for two *different* map databases and there are consequently few instances of different maps linked by common identifiers. It is more likely that branches of an organization agree upon a common map database, in which case the transfer of locations is trivial, and the only question is how well a field measurement agrees with a database measurement. Second, due to the accumulation of proportional error in field measurement, professional linear referencing practice employs intermediate reference points, control sections or anchor sections (e.g. Pennsylvania DOT, probably the most stringent, requires control sections to be no longer than 600 m); the LRP does not provide for these.

Despite these issues, the LR testing afforded an opportunity to quantify a number of aspects of accuracy of maps and linear measurements. There were three thrusts to the tests: (a) the accuracy with which a point acquired by GPS could be related to the road network, (b) comparison of linear measures using digitized length, GPS and DMI, (c) the accuracy with which known points could be transferred from one database to another using the LRP.

Accuracy of GPS Transfer

Many ITS services assume that a vehicle reports its position using GPS. Due to errors in the map database, and errors in the GPS reading, a GPS point does not always lie on the intended street in the map database (Figure 3), and the vehicle position may be reported incorrectly (although error can be reduced by map matching: see Taylor et al 2001). Additionally, the linear reference could be miscalculated.

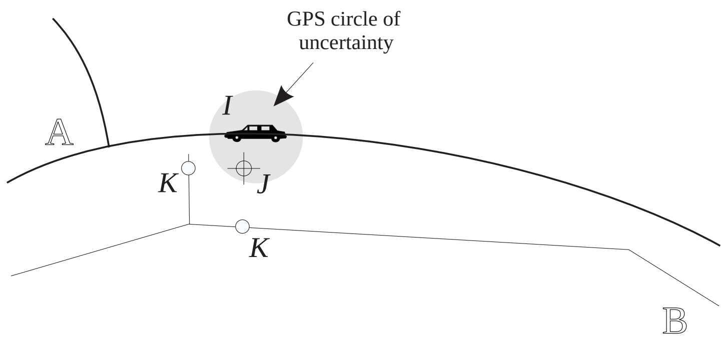

Figure 3. Map database A shows the true geometry of a road. A vehicle at I gets a GPS reading J, which snaps to K in a reference database (B). Clearly one instance of K is not the one intended.

The tests showed this to be a significant problem. GPS points were simulated by sampling points along an "engineering-scale" database, and generating random errors up to 30 m (GPS without Selective Availability error) and 100 m (GPS with S/A error). These coordinates were snapped to the nearest link in each test database. The point snapped to the wrong street or the wrong segment of the street 52% of the time on average, using low-cost commercial databases and 100 m GPS error. With 30 m GPS error, the failure rate was 25% — this is close to what can be expected today, with S/A turned off. With the engineering scale database, the likelihood of an error using 30 m GPS was 2–4%.

Comparison of Linear Measures

The results of the LR profile were predictable: in the case of the engineering database there was good correspondence between field measurements and geometrical measurements (1 in 1000), whereas with some commercial databases the polyline geometry was (a) generalized and (b) erroneous, leading to discrepancies as high as 15%.

The most surprising and useful finding from the LR tests was that coordinate tracks from differential GPS produced results comparable with a DMI. The discrepancy was in the region of 1%, which is approximately the same as the calibration accuracy of a DMI (note that the tests were conducted in 1998–99, and GPS S/A was turned off in May 2000; more recent VITAL research reported in the previous chapter showed discrepancies in the 0.1% to 0.5% range with *uncorrected* GPS). There are two important implications:

1. a centerline surveyed with differential GPS can serve as the basis for coordinate geometry measurements, with accuracy that is acceptable for the majority of applications;
2. given the accurate centerline, a point on a road (say the location of a speed limit sign) that is currently measured by LR can now be captured with GPS and converted to a linear reference, eliminating the laborious aspects of linear measurement in the field.

The length of a polyline surveyed by kinematic GPS is slightly dependent on vehicle speed (assuming that GPS is sampled at the best possible frequency, which is approximately every second). At higher speeds, the lateral error in the GPS reading is distributed over a longer segment, reducing the effect of the error.

Further study is required to determine the relationship between speed and accuracy, and the comparison of GPS corrected by various means (differential GPS, WAAS, polyline generalization) with DMI observations.

Validity of Message

There is a significant implementation issue with the LRP on freeways. The LRMS envisions automated composition and generation of messages, and on freeways the from- and to- intersections chosen by an unintelligent algorithm are inevitably intersections with ramps. Due to the low angle of incidence between a ramp and freeway, the intersection point is difficult to sight in the field, and there is uncertainty in placing it on a map; consequently significant discrepancies exist between databases, and terminal error up to 300 m can be introduced by this factor alone.

Concluding Points

Pursuant to our testing we were unable to recommend either the XSP or the LRP for general purpose use, either because the test databases (in 1997) were not of adequate quality to support the required interoperability objectives, or because the message specification made unrealistic assumptions. Outside the context of the LRMS design, it is clear from the tests that with reasonably good coordinate databases — achievable using current technology — the problems of LX for these applications are surmountable.

LRMS profiles are not a standard in highway asset management, and the errors reported in this chapter do not apply to transportation professionals who use roadside referencing infrastructure and more rigorous methods of LX.

Conclusion: Towards Reconciliation of LR and Coordinates

There have been significant changes in asset data management over the last two decades. In the 1970s, records on bridges, guard rails, potholes, traffic incidents and emergencies were largely paper-based. This gave way to computer databases in the 1980s. GIS was introduced, facilitating the integration of geographic context information, e.g. environment and climatic factors (fog, freezing, animal hazards); hydrology and flooding; geology and its implications for subsidence; population and travel demand forecasting. At the time, GIS treated transportation networks only as link-node structures, and every change of attribute along a link resulted in undesirable division of the link. The full potential of applications was therefore slow to develop. In the mid 1980s, dynamic segmentation methods were developed in GIS, the specialty area of GIS-T emerged, and the studies referenced in a previous chapter developed the linkages between LR and GIS-T.

In the meantime, DOTs were faced with increasing pressure to improve management systems. Efficiency and accountability became buzzwords of the 1990s. The Highway Performance Management System (HPMS) prompted agencies to organize all their operations more efficiently. This placed new demands on the amount of data to be gathered, maintained and processed.

We are now approaching a future where Intelligent Transportation Systems (ITS) create a quantum increase in information demand. Drivers are well oriented with GPS, on-board navigation systems and vehicle based wireless location based services. In-vehicle sensors measure distance to roadside objects with cm-level precision. DOTs are already in the process of introducing coordinate-based LX, and other methods that combine absolute and relative positioning methods are about to follow. What are the implications for their co-existence/reconciliation with LR?

Comparison

Even if positioning technology (GPS) had always been available, Linear Referencing would have existed alongside it as one of the principal LX methods for highway management. The experience of driving is linear in space and time; and in highway management, the most obvious way to partition a road for management purposes is by linear section. Even when a 2-d map of the road and one's real time progress along it are available, the linear conceptualization is useful, e.g. in judging ETA, or for quick mental calculations of road length. A cursory comparison of LR and coordinates (Table 1) shows that neither is inherently superior across all criteria. One could argue that the linear view of a road is "stripped down" and necessarily information-poorer than a multi-dimensional view. Yet it is clear that LR is here to stay and although the use of DMIs in the field will diminish slightly, the concepts and practice of LR will never be entirely replaced.

Table 1. Cursory comparison of linear and coordinate referencing

Criterion	Linear	Coordinates
Location eXpression elements	{Traversal ID, Traversal Reference Point, Linear Measurement}	{Latitude, Longitude} or {Road/Traversal ID, Latitude, Longitude}
Field equipment	Odometer (0.16 km) or DMI (0.001 km)*; accuracy depends on calibration	GPS (0.15 km), D-GPS (0.005 km) or WAAS-GPS (0.003 km) **
Principal advantage	Intuitive, easy to compare readings at different points on highway	Easy to gather field location data using GPS
Principal disadvantage	Field observation and maintenance of integrity are extremely laborious	Cannot process independently — require a reference map
Infrastructure required	Field reference markers; mileage equations or alternative;	Centerline reference file

*Resolution **Accuracy

Challenges of Co-existence

The obvious challenge of co-existence is to convert, easily and accurately, between the two forms of LX — and others such as street names, landmarks, temporal references (as in transit schedules), etc. An alternate approach is to break down the distinctions between the expressions, so that each strengthens the other. Ultimately this discussion must take place within a context of accuracy requirements, cost of accuracy and the operational benefits of accuracy (or cost of inaccuracy). These issues are discussed below, and a research and implementation agenda arises from this.

Mutualism

What can coordinate LX learn from LR?

In the case of coordinates, one of the principal problems is accuracy: current maps are geometrically inaccurate, and due to uncertainty in GPS, a point may appear to be on the wrong road. However, once the road is correctly identified, coordinates are sufficiently accurate for many purposes. Coordinate LX can borrow from LR, evolving the traditional coordinate expression

{Latitude, Longitude}

to

{Road_ID, Latitude, Longitude} or {Traversal_ID, Latitude, Longitude}.

Latitude and longitude are used generically here, and may be replaced with coordinates in UTM or other projections. This extended coordinate expression is now used in a few jurisdictions, most notably New York State DOT (Winters 2001).

What can LR learn from coordinate LX?

Similarly, the principal drawback of LR is that because it is a relative rather than absolute referencing system, it requires an elaborate network of field markers to serve as reference points for linear measurements. As upstream portions of the network change, marker properties need to be updated, i.e. the linear measure of each downstream reference point changes (in most jurisdictions, marker "269.25" no longer implies a measure of 269.25 from anything; it is for practical purposes an ordinal label; nonetheless, the linear reference of the marker has to be updated periodically to establish overall location). A solution is to specify reference points in absolute terms, i.e. coordinates. Unfortunately data models such as 20-27(2) pursue linear measurement to the exclusion of any form of absolute (coordinate-based) spatial orientation; this approach does not equip the user for a future that is inescapably hybrid.

California's network of "post-mile" markers are well surveyed points with 3-d information associated with each point, and are thus equipped for hybrid referencing; their density is sufficient for approximately ±8 m linear referencing accuracy using ±1% DMI technology — this is comparable to the accuracy achievable using consumer grade uncorrected GPS receivers.

Conversion — Methods and Accuracy

The methods for conversion between linear and 2-d coordinates were discussed earlier (page 4). At the level of algebra, the methods are well defined. The problem is that they do not work well where centerline geometry is inadequate. Methods for improving centerline geometry were discussed above (page 4).

What will it take in terms of geometric centerline accuracy, for LR to be fully compatible with coordinate LX? It is important to understand that the two can never be 100% compatible, because neither road length nor coordinate representation of a road is 100% definable. In a previous chapter we have referred to an "engineering scale" database, but this phrase is only loosely associated with a numerical accuracy standard, about ±1 m of horizontal accuracy. In practice one has to establish and to enforce an error tolerance in functional terms, e.g. DMI length must agree with digitized length ±1%. This in turn must be tied to information accuracy requirements.

Information Accuracy Requirements

To establish information accuracy requirements, it is necessary to examine the need for and cost of accurate information, and the nature of decisions that rely on it. The start and end of a no-passing zone must be specified with relatively high accuracy because of the potential for litigation in the event of an accident. The location of an informative "No services" sign is less critical, and it is not cost-effective to enforce stringent "engineering" accuracy requirements for such asset locations. On the other hand there is a cost associated with having multiple requirements for different types of assets. A research issue that has not been adequately addressed in the general literature is how accurate information needs to be to support decision making, the technologies associated with these accuracies, and the costs associated with the technologies (NCHRP project 20-47 was intended to address this; outside transportation the issue is treated in the GIS literature and some consulting reports).

Closing Comments

This report has laid out a set of systematic arguments for a broader view of Location eXpression in DOT operations, precipitated by needs in data sharing and data integration. Consumer grade GPS now offers accuracy comparable with that from DMIs, and this presents a useful alternative to linear referencing for some DOT operations. This does not mean that LR is obsolete, but that it will increasingly need to be complemented with other forms of LX, and that operational accuracy requirements based on decisions should govern specifications and methods. The arguments presented here are broad, and supported by technical research already published (VITAL 2000) and in progress.

There are numerous initiatives in the U.S. and worldwide to create accurate centerline geometry. The projects are being pursued because of rapidly changing technologies and the growing need for accurate positioning in DOT operations. There is a danger that highly accurate centerline geometry will be pursued independently of a careful assessment of functional requirements, and that funds will be over-expended on excessive accuracy, or under-expended on insufficient accuracy. For this reason it is important that the relationships in the previous section (Conversion — Accuracy) be researched, and that the independent objectives of "faster, cheaper, better" each be associated with appropriate methodologies and recommended practice.

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